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ONTARIO

DEPARTMENT OF EDUCATION

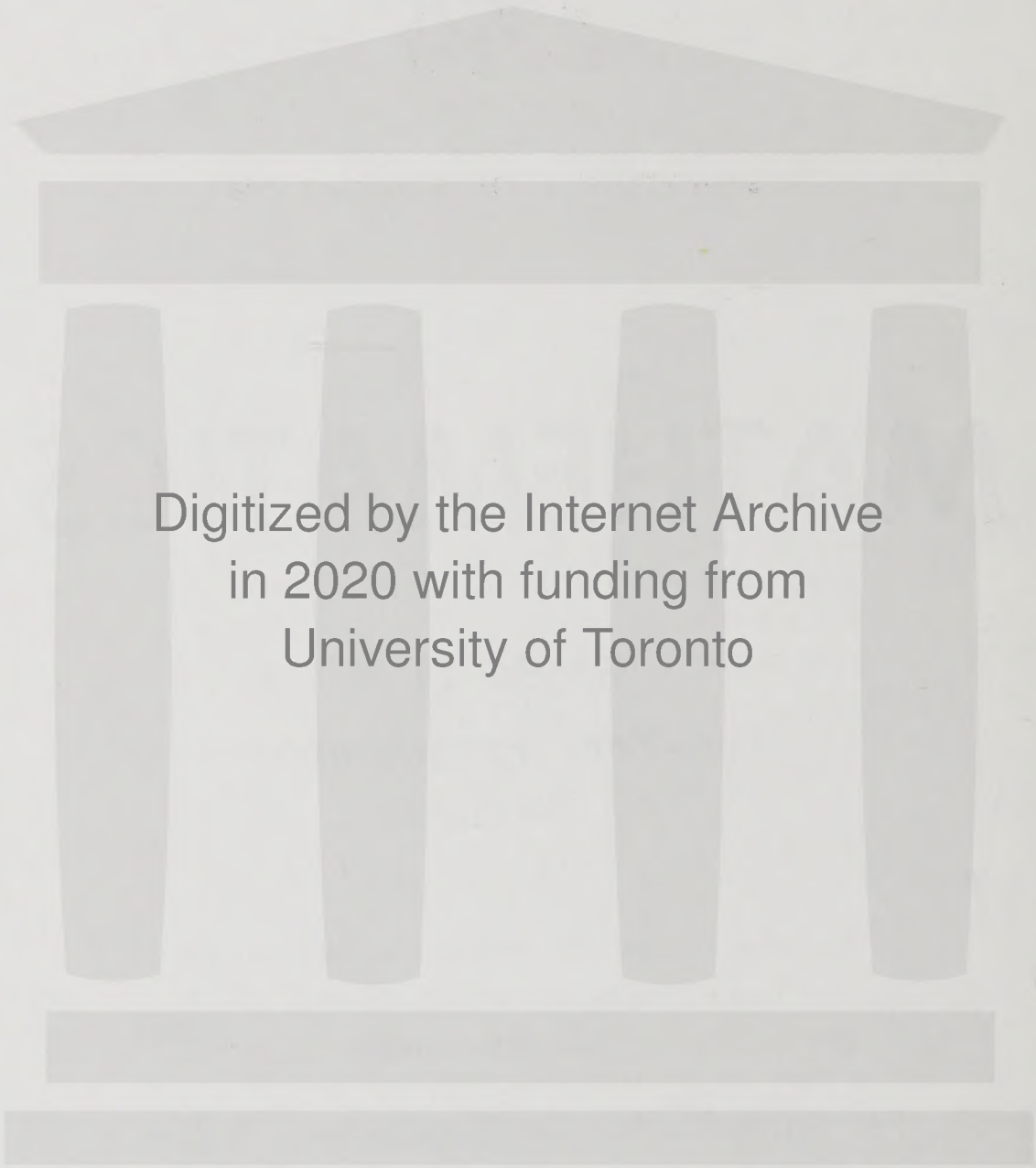
MATHEMATICS

Grade 11
Five-Year Programme
All Branches

Replacing the course in Mathematics for Grade 11
as printed in Curriculum S.12 (1961)
Senior Division Mathematics

For introduction in September, 1964

ISSUED BY AUTHORITY OF
THE MINISTER OF EDUCATION



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MATHEMATICS
Grade 11

1. The course for Grade 11 outlined below represents the first step in a revision of the courses for mathematics in the Senior Division of all five-year programmes in the Ontario Secondary Schools. Commencing in the school year 1964-65, it will replace the Grade 11 section (pages 5 to 14) of Curriculum S.12 "Senior Division Mathematics".

   The second and third steps in the revision will go into effect in Grade 12 in 1965-66, and in Grade 13 in 1966-67. Outlines of the courses for Grades 12 and 13 will be given in later releases.

2. An examination of this course outline will reveal that the actual topics listed are not basically different from those now listed at some level of the Senior Division.

   The greatest change is that the topics are so arranged as to make this a course in mathematics, rather than of algebra, geometry, or trigonometry. This is achieved by making the whole course a study of the straight line, either as a single straight line or in rectilinear figures, and bringing to bear thereon whatever branch of mathematics will serve this study.

   In the topics where the language of the set theory adds clarity or conciseness, some change in approach and language is indicated.

3. Pupils who have completed successfully the Grades 9 and 10 mathematics outlined in Curriculum I.1 of 1951 will have the necessary prerequisites for the Grade 11 course as revised.

   Pupils in schools that have used the optional approach as suggested in RP12 Mathematics (at the top of page 2) should be able to progress more rapidly in the earlier units of the course. To provide for this situation, some supplementary topics are suggested for enrichment, and it is expected that teachers will add more if time allows.

4. It is most important in an integrated course of this nature that all topics indicated as compulsory must be covered. Careful attention should, therefore, be paid to the number of weeks suggested for each unit. These times have been carefully chosen not only to indicate time-planning limits but also to indicate depth of treatment.

5. It is intended that this will remain as the prescribed course until the results of changes in courses in lower grades will make further changes advisable in the Senior Division.

UNIT 1:
NUMBER SYSTEMS AND NUMBER-LINES (6 weeks)

GENERAL COMMENTS

1. This unit serves to acquaint pupils with a knowledge of the number systems up to and including real numbers. It covers and expands part of section 4 of the present Grade 12 course.

2. In this topic, one can make use of some of the language and concepts of sets. Some sections on sets are, therefore, included at the appropriate places. It is to be understood that this will be a new experience for those pupils who have been following Curriculum I.1 (1951) through Grades 9 and 10, and the development must take account of this fact.

OUTLINE FOR THIS UNIT

1. Set, subset, null set; some of the simple symbols used

   Practice in conversion of words to symbols and vice versa

   Some simple examples of unions and intersections of sets

2. The natural numbers and zero

   The closure, commutative, associative, and distributive properties

   The simple properties of 0 and 1

   Limitations on the operations, including inadmissibility of division by zero

Determination of solution sets of simple equations and inequalities which can be solved in the system of natural numbers and zero

The set-builder notations

Graphing on a line

3. The integers; simple properties of the integers

Absolute value

4. The rational numbers

Study of the reversibility of the change of repeating and terminating decimals to the form $\frac{a}{b}$

An inductive development to show that the rationals are everywhere dense

5. The introduction of the real numbers from a study of examples of non-terminating, non-repeating decimals

An inductive demonstration to show that these are real non-rational numbers with a place on a number-line

Reference to special occurrences of these numbers (π, roots, etcetera)

A discussion of the order relation

(Supplementary) The formal proof that $\sqrt{2}$ is not rational

6. A recapitulation of the number systems through the natural numbers and zero, the integers, the rationals, and the reals, to show

(a) closure limitations

(b) the growing complexity of the problems which can be solved

7. Determination of the solution sets of equations and inequalities (Emphasize equivalent equations and inequalities)

Graphing on the real number line

(Note that this will be the first experience with formal solution of inequalities for many pupils)

8. Operations with radicals, including rationalization of denominators

   Equations involving radical signs which give rise to linear equations in one variable,

   e.g., $\sqrt{x-3} = 2$; $\sqrt{5x+1}-2\sqrt{x+3} = 0$;

   $$\frac{\sqrt{x}+16}{\sqrt{x}+4} = \frac{\sqrt{x}+29}{\sqrt{x}+11}, \text{ etc.}$$

UNIT 2:

GEOMETRY: CONGRUENCE, PARALLELISM

(3 weeks)

GENERAL COMMENTS

1. This unit forms a review of the Grade 10 deductive geometry of congruence and parallel lines with an extension into work on loci. It is expected that most of the time allowed will be devoted to the analysis and solution of deductions. The only new element is some increase in the use of algebraic designations and symbols.

2. It is important to emphasize applications and deductions based upon the theorems. It is unnecessary to re-prove theorems already proved in Grade 10.

3. A good part of each lesson period in the geometry of this unit and the following units should be devoted to the ORAL presentation of solutions of deductions by the students, using large, well-marked, black-board diagrams. Demonstration should be followed by class criticism, discussion, and alternative solutions.

   The ability to WRITE a proper solution for a deduction is important also. However, it is not expected that full written solutions for all assigned deductions will be placed in a work-book.

OUTLINE FOR THIS UNIT

1. A review of the descriptions of the common geometrical figures (using the language of sets where appropriate)
   (a) The use of the "undefined term" approach for the nouns: set, point, line, curve, plane, space

(b) Intuitive approach to the descriptions of: plane, line, line segment, ray, angle, triangle, simple closed curve including simple polygons, region

2. Measurement of line segments and angles

   Association of a real number with each line segment or angle

   A classification of angles and triangles according to measures of line segments or angles

3. Deductions involving angles, triangles, quadrilaterals, and parallel lines (including inequalities)

4. Locus as a set of points and as path of a moving point

   Review of the standard constructions of bisecting angles and lines using the locus concept

   Extension of the locus concept where two or more independent conditions are to be satisfied (incentre, circumcentre, etc.)

5. (Supplementary)

   Some simple three-dimensional geometry, including the study of intersections of lines and planes, planes and planes.

   Practice at representation of three-dimensional figures and of lines or planes in space

   Study of unions and intersections of planes in prisms and pyramids

   Inductive discovery of the Euler relation, $f+v = e+2$

   Proof that there are only five regular convex polyhedra

UNIT 3:
AREAS AND THE PYTHAGOREAN RELATION
(3 weeks)

GENERAL COMMENTS

1. This unit is set up as a review of the theorems on area that will have been taught in Grade 10.

2. In covering section 1 of this unit, the alternate order, i.e., (b), (c), (a), (d), may be used.

OUTLINE FOR THIS UNIT

1. (a) Proofs and geometrical constructions relating to areas of triangles and parallelograms with equal or same bases
   (b) Assumption regarding area of rectangle
   (c) Development of areas of parallelograms and triangles in terms of base and altitude
   (d) Algebraic and arithmetic problems relating to areas of triangles and parallelograms
2. Proof of the Pythagorean relation using areas (It is not expected that this proof will be required for test purposes.)
   Applications of the Pythagorean relation in triangles
3. Calculations of the area of triangles using the Heron formula (The proof of this formula is a supplementary topic.)
4. Lateral and total areas of prisms and pyramids

UNIT 4: RATIO AND PROPORTION (5 weeks)

GENERAL COMMENT

The aim of this unit is to develop the use of ratios in algebraic problems and with rectilinear figures. Accordingly, it uses part of Book 4 and some of the topics previously covered in Grade 12 algebra.

OUTLINE FOR THIS UNIT

1. (a) The meaning of ratio
   Ratio developed as an ordered doublet or triplet or n-tuplet of numbers not all zero
   Use of the colon notation
   (b) Problems from arithmetic, algebra, science, and mensuration to give practice in writing ratios involving two or more terms, and in reducing ratios
   (c) Equivalence of ratios
   Solution of problems involving proportion

(d) Special study of two-term ratio
Condition for equivalence of two-term ratios
Mean proportional

Practice at transformation of proportion involving inversion, alternation, adding and subtracting unity
Correspondence between the ratio a:b (b not equal to zero) and the rational number $\frac{a}{b}$

(e) (Supplementary) Division of a line segment in a given ratio internally and externally
Problems thereon

2. Proofs and problems based on the following theorems and constructions

(a) Areas of triangles of equal altitudes (bases) are proportional to their bases (altitudes)

(Note: It is suggested that some of the problems thereon make use of the identity method

$$\frac{a}{b} = \frac{a}{c} \times \frac{c}{d} \times \frac{d}{b}, \text{ etc.})$$

(b) A straight line drawn parallel to one side of a triangle divides the other two sides in the same ratio (and the converse)

(c) If two triangles are equiangular, their corresponding sides are proportional (and the converse)

(d) If two triangles have an angle of one equal to an angle of the other, and the sides about these angles proportional, the remaining corresponding angles are equal and hence the triangles are similar

(e) The areas of similar triangles are proportional to the squares on the corresponding sides

(f) The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides (It is suggested that this proof using ratio, rather than the former proof using area, be emphasized)

(g) The construction of a line segment whose length is the mean proportional between the lengths of two given line segments

Note: It is suggested that every mathematics classroom library include several copies of standard synthetic geometry texts. The better students should be encouraged to do extensive supplementary work from such texts so that the particular methods of this specific discipline will be fully realized by these pupils.

UNIT 5: TRIGONOMETRY (4 weeks)

GENERAL COMMENT

This unit aims to give a short introduction to trigonometry as a natural corollary to the work on ratio and proportion.

OUTLINE FOR THIS UNIT

1. Angles in degrees and radians
   (Simple conversions only)

2. Review of the Grade 10 topic of co-ordinates and plotting in the real number plane

3. Sine, cosine, and tangent of an angle using general definitions $\frac{x}{r}, \frac{y}{r}, \frac{y}{x}$

   Proof that values for any given angle are independent of r

   Practice determining values for angles with terminal ray in various quadrants

4. Definitions of cosecant, secant, cotangent
   Development of the reciprocal relations
   Development of the Pythagorean relation
   Simple identities

5. Practice determining ratios for right-angled triangles with the right angle in various positions
   Complementary relations

6. Geometrical determination of the trigonometric ratios for $\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}$

   Discussion of values for 0 and $\frac{\pi}{2}$

   Use of tables (four decimal figures for one degree intervals) to determine values where x is the measure of the angle and $0 < x < \frac{\pi}{2}$

7. Solution of right-angled triangles
   Practical problems including angle of elevation and angle of depression

8. (Supplementary)

   (a) Development of the sine and cosine laws for acute-angled triangles
   Simple applications of these laws

   (b) A study of projections and the theorems thereon
   Connections with the cosine law

   (c) Solution of triangles in three-dimensional configurations

UNIT 6:
LINEAR EQUATIONS AND INEQUALITIES (4 weeks)

GENERAL COMMENT

The aim of this unit is to extend the work on solution of equations taken in Grade 10, and to study the graphical implications of the solutions. Graphical implications of the solution sets for inequalities, as well as limitations of the domain and range, are included.

OUTLINE FOR THIS UNIT

1. Ordered pairs; correspondence between ordered pairs and points in the Cartesian plane
2. Domain and range; stress of the real domain
3. Graphs of solution sets for the first degree equations and inequalities

Inductive development of the fact that, in general, the graph of a linear equation is a straight line and the graph of an inequality is a region

Graphing solution sets for systems involving first-degree equations and inequalities

4. Solving a system of two linear equations in two variables using the method of equivalent systems

   Graphical implications of the equivalent system method showing the graphical stage reached at each step

   A review of the methods of algebraic solution: comparison, substitution, elimination

   Word problems solved by systems of linear equations

   Classification of systems of linear equations as consistent, dependent, inconsistent; graphical implications

   Systems of linear equations with literal coefficients

   Systems of three linear equations in three variables

   Word problems involving these

5. (Supplementary)

   (a) Diophantine equations arising from problems; types of diophantine equations; number of solutions

   (b) The graphical interpretation of a linear equation in three variables

   (c) The general system of two equations of the first degree in two variables

   General conditions for systems that are consistent, dependent, inconsistent

UNIT 7: ANALYTIC GEOMETRY (4 weeks)

GENERAL COMMENT

The aim of this unit is to introduce some of the fundamentals of analytic geometry of the straight line.

OUTLINE FOR THIS UNIT

1. Distance between two points

   Check of collinearity by means of distances

2. Midpoint of a line segment
   Internal division of a straight line
   (Supplementary) External division

3. The concept of slope
   Calculation of slope (Also slope equal to $\frac{\triangle y}{\triangle x}$ and to $\tan \theta$)

4. Finding the equation of a straight line in point-slope form, including the cases of the slope y-intercept form and the slope x-intercept form

5. Condition for parallelism and perpendicularity of two straight lines

6. (Supplementary)
   (a) Area of a triangle
   Test for collinearity by area
   Distance of a point from a line

   (b) Some proofs by both analytic and synthetic methods, e.g.,
      (i) A line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to one-half of it
      (ii) The diagonals of a rhombus are perpendicular
      (iii) If two medians of a triangle are equal, the triangle is isosceles

   (c) The alternative proof of the law of cosines by the use of coordinates